647510 **BSc5MatC501x** **Seat No :________**

B.Sc. Semester - 5 (*CBCS*) Examination
Oct/Nov - 2025 (NEW COURSE)
Mathematical Analysis -1 & Abstract Algebra -1(Core)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1(A) Answer any two out of three. (04)

(1) State and prove Housedorff property.

(2) X is a metric space. Show that intersection of finite numbers of open subsets of X is open. Explain with example that this statement is not true for infinite numbers of open subsets of X.

(3) Show that the cantor set C is closed and prove that $\frac{9}{10} \in C$.

Que.1(B) Answer any one out of two. (10)

(1) d is define on R as; for $\forall$ x,y$\in$R

$$d(x,y) = \begin{cases} 0 & ; \quad x=y \\ \sqrt{x^2+y^2} & ; \quad x \neq y \end{cases}$$

Then show that (R,d) is a metric space.

(2) Construct set of real numbers which has exactly three limit points.

Que.2(A) Answer any two out of three. (04)

(1) Function f is bounded in [a,b]. P and P^* be two partitions of [a,b] Such that $P \subset P^*$ then show that $L(P^*,f) \geq L(P,f)$.

(2) If functions f and g are R-integrable in [a,b] then show that f+g is also R-integrable in [a,b] and $\int_a^b [f(x)+g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$.

(3) Prove that any bounded function f defined on [a,b] is R-integrable iff For $\forall\ \varepsilon>0$ there exist partition P of [a,b] such that U(P,f) - L(P,f) < ε, Where $\|P\| < \delta$.

Que.2(B) Answer any one out of two. (10)

(1) Find $\int_1^2 (2x+1)dx$ using definition of R-integration.

(2) if function f is Monotonic in [a,b] then show that f is R-integrable in [a,b].

Que.3(A) Answer any two out of three. (04)

(1) If bounded function f is R-integrable in [a,b] and $F(x)= \int_a^x f(t)dt$, where $a \leq x \leq b$ then show that F(x) is continuous in [a,b].

(2) State and prove general form of first mean value theorem for R-integration.

(3) G=R-{-1}. Binary operation * is defined on G as ; for a,b$\in$G , a*b=a+b+ab then show that (G,*) is group and find 2^{-1} in G.

Que.3(B) Answer any one out of two. (10)

(1) If a,b ≠ 0, a>b then show that $\frac{\pi^2}{2a} < \int_0^{\pi} \frac{x}{a\cos^2\frac{x}{2} + b\sin^2\frac{x}{2}}dx < \frac{\pi^2}{2b}$

(2) Show that in group G, for $\forall$ a,b$\in$G, equation a*x=b has a unique solution in G.

Que.4(A) Answer any two out of three. (04)

(1) Prove that order of permutation $f \in S_n$ is L.C.M. of lengths of disjoint cycles of f.

(2) G is finite group and H≤G. Show that O(H)/O(G).

(3) Define alternating group and prepare group table for alternating group of the symmetric group S_3 in usual notations.

Que.4(B) Answer any one out of two. (10)

(1) G is finite group. For a,b $\in$ G Show that $O(b^{-1}ab)=O(a)$.

(2) $\sigma \epsilon$ S_6 where $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 4 & 2 & 1 & 3 & 5 \end{pmatrix}$ then find σ^{-1} and $o(\sigma)$. Also check whether σ is even or odd.

Que.5(A) Answer any two out of three. (04)

(1) G is finite group and a ϵ G. prove that O(a)/O(G).

(2) State and prove Cayley's theorem.

(3) if $(G,*) \cong (G',*')$. Show that G is commutative iff G' is Commutative. where G and G' are groups.

Que.5(B) Answer any one out of two. (10)

(1) For $n \geq 2$ show that A_n a is normal subgroup of S_n.

(2) Prepare group table in usual notation for quotients group $Z_6/<3>$. Where $(Z_6,+_6)$ is a group.

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